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Statistical properties of the output of linear amplifier with superpositions of squeezed displaced Fock states as an input state

G.M. Abd Al-Kader[a]

Mathematics Department Faculty of Science, Al-Azhar University, Nasr City, Cairo, Egypt

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Abstract. The s-parameterized characteristic function for the output field with the superposition of squeezed displaced Fock states (SDFS's) as input field is given. The s-ordered distribution functions for the output field with superposition of SDFS's as input state are investigated. Various moments are calculated by using the s-ordered characteristic function for that field. The Glauber second-order coherence function is calculated. The quadrature squeezing for the output field are discussed. Some Quasiprobability distribution functions of the output fields are plotted as functions of the interaction time. The quadrature squeezing for the output field are discussed.

PACS. 42.50.Dv Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements – 42.50.Vk Mechanical effects of light on atoms, molecules, electrons, and ions – 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov Bohm effect, Bell inequalities, Berry's phase)

1 Introduction

There has been vigorous investigations for the linear interaction of atoms with an optical field in the last couple of decades [1,2]. A simple model for linear phase-insensitive amplification or absorption can be obtained by considering the medium to be a collection of two-level atoms with populations N_2 and N_1 in the excited and ground states respectively [1c]. When a signal passes through an amplifier, it is modified in two ways: First the amplitude of the signal is amplified. Second noise is added to the signal. The noise can be either phase dependent or phase independent. Here the quantum statistical properties of the output is considered to be depending on the quantum statistical properties of the input field.

The quasiprobability distribution function is a c-number functions, not necessarily positive, that allows one to calculate the expectation values of a quantum system [3,4]. Recently the quasiprobability functions have become accessible to experimental measurement by means of the optical homodyne technique. These measurement schemes have revealed a new facet of the s-paramaterized quasiprobability functions in coherent state basis, namely that the s-parameterized quasiprobability distribution function with fractional values of s ($|s| < 1$) is what is actually seen by the detectors. The value of the parameter s as revealed by these experiments is directly related to the detector efficiency and the amplification of the laser amplifier used in these schemes [5].

The squeezed displaced Fock states (SDFS's) have been introduced and studied in [6]. These states generalize two-photon coherent states [7] (squeezed coherent states), squeezed number states [8], and displaced Fock states [6,8]. They exhibit both number squeezing in the strong sense and the quadrature squeezing. Recently the creation of nonclassical states of motion of a trapped ion such as Fock states, coherent states, squeezed states and Schrödinger cat states have been reported [9]. That moved the study of these states from the academic realm to the world of experimentation. This motivated us to study the linear amplifier superposition of these states.

Superposition of quantum-mechanical states of electromagnetic field have recently received much attention in quantum optics [10–14], since these states can exhibit non-classical properties of light, such as quadrature squeezing and sub-Poissonian photon statistics. In particular, the Schrödinger cat states, are superpositions of distinguishable macroscopic quantum states of a single mode of the quantized electromagnetic field and are usually given as superpositions of two ordinary coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, which are separated in phase by 180° [10]. The linear superpositions of a finite number of SDFS's has been studied [15]. The purpose of this article is to study the quantum statistical properties of the output field with superposition of SDFS's as input state.

[a] e-mail: obada@frcu.eun.eg

This paper is organized as follows. In Section 2, we discuss the s-parameterized characteristic function on a linear amplifier with superposition of a pair of SDFS's as input state. We study some applications for the s-ordered characteristic function: namely, moments and squeezing. In Section 3, we discuss the s-parameterized quasi-probability distribution function. We conclude the paper in Section 4 with some brief remarks.

2 s-parameterized characteristic function on a linear amplifier with SDFS's superposition state as input field

We briefly discuss an optical linear amplifier and its dynamics. We assume that there exist N_T two-level atoms concentrated in a very small region of the space compared with the radiation wavelength, and that a single-mode of the electric field interacts with their dipole moments through the atomic transitions. The field frequency is resonant with the atomic transition frequency and the position-dependent variable of the field is eliminated. Suppose that N_1 of the atoms are in the lower state and N_2 in the upper state ($N_T = N_1 + N_2$). The system behaves as an amplifier if $N_1 < N_2$, and as a field attenuator when $N_1 > N_2$. The density operator ρ of the field obeys the following equation, [1]

$$\frac{\partial \rho}{\partial t} = \eta N_2(2a^+\rho a - aa^+\rho - \rho aa^+) + \eta N_1(2a\rho a^+ - a^+a\rho - \rho a^+a), \tag{2.1}$$

where a and a^+ are the usual single-mode photon annihilation and creation operators and η denotes the coupling constant between the atoms and the field.

According to Cahill and Glauber [3] the P (Glauber-Sudarshan), W (Wigner) and Q (Husimi) functions may be expressed in an integral form

$$F(\alpha, s) = \frac{1}{\pi^2}\int C(\beta, s)\exp(\beta^*\alpha - \beta\alpha^*)d^2\beta \tag{2.2}$$

where $C(\beta, s)$ is the s-ordered generalized characteristic function

$$C(\beta, s) = Tr[D(\beta)\rho]\exp(\frac{s}{2}|\beta|^2) \tag{2.3}$$

and s is a parameter which defines the relevant quasiprobability distribution functions. For $s = 1$ we obtain the Glauber-Sudarshan P-function, for $s = 0$ we have the Wigner function, and for $s = -1$, we have the Q-function. In equation (2.3), $D(\beta)$ is the Glauber displacement operator [7] (see (2.11) below), and ρ is the density matrix of the field under investigation.

From equation (2.1) Carustto [1c] was able to find the normally ordered characteristic function, $C_N(\zeta, t)$, which is defined as

$$C_N(\zeta, t) = Tr[\rho(t)\exp(\zeta a^+)\exp(-\zeta^* a)]. \tag{2.4}$$

It is found that

$$C_N(\zeta, t) = C_1(\zeta, t)C_2(\zeta, t) \tag{2.5}$$

where

$$C_1(\zeta, t) = Tr[\rho(0)\exp(G^*\zeta a^+)\exp(-G\zeta^* a)] \tag{2.6}$$

and

$$C_2(\zeta, t) = \exp[-\frac{N_2}{N_2 - N_1}(|G|^2 - 1)|\zeta|^2] \tag{2.7}$$

with

$$G(t) = \exp[\eta(N_2 - N_1)t - i\omega t] \tag{2.8}$$

where $\rho(0)$ is the initial density operator, and ω the frequency of the field. So that the system is an amplifier if $N_2 > N_1$. The quantity $|G|^2$ is the gain of the amplifier, in fact $|G|^2$ will be the gain ($N_2 > N_1$) or loss ($N_1 > N_2$) factor.

For an input field on the linear amplifier, we choose a superposition state $|\Psi_m\rangle$ which is assumed in the form

$$|\Psi_m\rangle = A^{-\frac{1}{2}}\{|\alpha_0, z, m\rangle + K|-\alpha_0, z, m\rangle\} \tag{2.9}$$

where $|\alpha_0, z, m\rangle$ is SDFS given by

$$|\alpha_0, z, m\rangle = D(\alpha_0)S(z)|m\rangle \tag{2.10}$$

where the displacement operator $D(\alpha_0)$, (with $\alpha_0 = |\alpha_0|\exp(i\theta_0)$ a complex parameter that represents the magnitude and angle of the displacement), and squeeze operator $S(z)$ are given by [7]

$$D(\alpha_0) = \exp(\alpha_0 a^+ - \alpha_0{}^* a), \quad S(z) = \exp\left[\frac{z^*}{2}a^2 - \frac{z}{2}a^{+2}\right], \tag{2.11}$$

where $z = re^{i\phi}$ and r is known as the squeeze parameter and ϕ indicates the direction of squeezing. With A is the normalization constant given by

$$A = \{1 + |K|^2 + (K + K^*)e^{-2|\bar{\alpha}_0|^2}L_m(4|\bar{\alpha}_0|^2)\} \tag{2.12}$$

where $\bar{\alpha}_0 = \mu\alpha_0 + \nu\alpha_0{}^*$, $\mu = \cosh r$, $\nu = \exp(i\phi)\sinh r$ and $L_m^\sigma(x)$ is the Laguerre polynomial

$$L_m^\sigma(x) = \sum_{s=0}^{m}\binom{m+\sigma}{m-s}\frac{(-1)^s}{s!}x^s. \tag{2.13}$$

For $K = 0$ we have the SDFS's, but for $K = 1$ or -1 the resulting states depend on m: if m is an even number and $K = 1$, we have superposition of even states and while odd states are obtained when $K = -1$. But when m is an odd number the result, is reversed.

The density operator for an input state of the single-mode field given by the superposition of a pair of SDFS's, takes the form

$$\rho(0) = |\Psi_m><\Psi_m|. \tag{2.14}$$

By using the operators identities, we can write the s-ordered characteristic function in the form,

$$C(\zeta,s,t)=\frac{1}{A}\exp\Big[\Big\{\frac{s-1}{2}+\frac{1}{2}|G|^2-M(t)\Big\}|\zeta|^2\Big]$$
$$\times\Big\{\Big[\exp\Big\{-\frac{1}{2}|\bar{\zeta}|^2\Big\}L_m[|\bar{\zeta}|^2]\Big]\Big[\exp\big[G^*\alpha_0^*\zeta-G\alpha_0\zeta^*\big]$$
$$+|K|^2\exp\big[-G^*\alpha_0^*\zeta+G\alpha_0\zeta^*\big]\Big]+K\exp\Big\{-\frac{1}{2}|\bar{\zeta}-2\bar{\alpha}_0|^2\Big\}$$
$$\times L_m[|\bar{\zeta}-2\bar{\alpha}_0|^2]+K^*\exp\Big\{-\frac{1}{2}|\bar{\zeta}+2\bar{\alpha}_0|^2\Big\}L_m[|\bar{\zeta}+2\bar{\alpha}_0|^2]\Big\} \quad (2.15)$$

where

$$\bar{\zeta}=\mu G^*\zeta+\nu G\zeta^* \quad (2.16)$$

and

$$M(t)=\frac{N_2}{N_2-N_1}(|G|^2-1). \quad (2.17)$$

Thus the s-parameterized CF is obtained; and from it we can calculate any expectation value for the field operators.

2.1 Moments

We calculate the moments of the photon operators for the output of linear amplifier with superposition of SDFS's as an initial state. We also present the average values of the quadrature operators. The s-ordered average value of a and a^+ can be calculated in the following way

$$\langle[a^{+k}a^l]_s\rangle=Tr[\rho\{a^{+k}a^l\}_s] \quad (2.18)$$
$$=\frac{\partial^k}{\partial\zeta^k}\frac{\partial^l}{\partial(-\zeta^*)^l}C(\zeta,s,t)|_{\zeta=\zeta^*=0} \quad (2.19)$$

or through an integration involving the function $F(\beta,s)$.

The average values of the annihilation and creation operators are derived by differentiating the characteristic function equation (2.15) with respect to ζ and $-\zeta^*$, respectively:

$$\langle a^+\rangle=\frac{G^*}{A}\Big\{(K-K^*)\Big[2(\mu\bar{\alpha}_0^*+\nu^*\bar{\alpha}_0)\exp[-2|\bar{\alpha}_0|^2]$$
$$\times L_{m-1}^1(4|\bar{\alpha}_0|^2)+\alpha_0^*\exp[-2|\bar{\alpha}_0|^2]L_m(4|\bar{\alpha}_0|^2)\Big]$$
$$+\alpha_0^*(1-|K|^2)\Big\}=(\langle a\rangle)^*. \quad (2.20)$$

Similarly,

$$\langle a^+a^+\rangle=\frac{G^{*2}}{A}\Big\{(K+K^*)\Big[4(\mu\bar{\alpha}_0^*+\nu^*\bar{\alpha}_0)^2\exp[-2|\bar{\alpha}_0|^2]$$
$$\times L_{m-2}^2(4|\bar{\alpha}_0|^2)+\{4(\mu\bar{\alpha}_0^*+\nu^*\bar{\alpha}_0)^2-2\mu\nu^*\}\exp[-2|\bar{\alpha}_0|^2]$$
$$\times L_{m-1}^1(4|\bar{\alpha}_0|^2)+\{(\mu\bar{\alpha}_0^*+\nu^*\bar{\alpha}_0)^2-\mu\nu^*\}\exp[-2|\bar{\alpha}_0|^2]$$
$$\times L_m(4|\bar{\alpha}_0|^2)\Big]+(1-|K|^2)[\alpha_0^{*2}-2\mu\nu^*m-\mu\nu^*]\Big\}=(\langle aa\rangle)^*. \quad (2.21)$$

The average number of photons can be acquired analogously:

$$\langle[a^+a]_s\rangle=\frac{|G|^2}{A}\Big\{(K+K^*)\Big[-4|\mu\bar{\alpha}_0+\nu\bar{\alpha}_0^*|^2$$
$$\times\exp[-2|\bar{\alpha}_0|^2]L_{m-2}^2(4|\bar{\alpha}_0|^2)+\{(|\mu|^2+|\nu|^2)-4|\mu\bar{\alpha}_0+\nu\bar{\alpha}_0^*|^2\}$$
$$\times\exp[-2|\bar{\alpha}_0|^2]L_{m-1}^1(4|\bar{\alpha}_0|^2)+\{|\mu\bar{\alpha}_0+\nu\bar{\alpha}_0^*|^2+A_1\}$$
$$\times\exp[-2|\bar{\alpha}_0|^2]L_m(4|\bar{\alpha}_0|^2)\Big]+(1-|K|^2)[|\alpha_0|^2$$
$$+m(|\mu|^2+|\nu|^2)-A_1]\Big\}, \quad (2.22)$$

where

$$A_1=\frac{-1}{|G|^2}\{\frac{1-s}{2}+|G|^2|\nu|^2+M(t)\}, \quad (2.23)$$

and $\langle a^+a^+aa\rangle$ can be analogously calculated.

The Glauber second-order coherence function is defined by

$$g^{(2)}=\frac{\langle a^{+2}a^2\rangle}{\langle a^+a\rangle^2}\cdot \quad (2.24)$$

It has been classified that the light with $g^{(2)}<1$ is a sub-Poissonian light, the light with $1<g^{(2)}<2$ is a super-Poissonian light, and the light with $g^{(2)}>2$ is called super thermal light. It is well known that the coherency is unity for the coherent light (Poissonian light).

Substitution of $\langle a^+a^+aa\rangle$ and (2.22) into (2.24) yields the coherence function for the superposition of SDFS's.

We plot autocorrelation function of equation (2.24) in Figure 1. In the figure the vertical axis measures the magnitude of coherency, while the horizontal axis indicates the interaction time t. We assume the parameters as follows: the number of photons $m=0$, 1, 2, 3 the displacement parameter $\alpha_0=1$, the squeeze parameter $r=0.5$ and the direction of squeezing is zero. The constant K has the values (a) $K=1$, (b) $K=i$, (c) $K=-1$. From Figure 1a, ($K=1$), we note the sub-Poissonian light exists for odd number of photons ($m=1,3$) in the SDFS's. The series in this case contains odd states only. The super-Poissonian statistics exist with $m=0$, 1, 2, 3 as t develops. Even for the case of light initially starting with sub-Poissonian distribution ($m=0$) turns to super-Poissonian as t develops in Figure 1. From Figure 1c, ($K=-1$), we note the sub-Poissonian light exists for even $m=0,2$ in SDFS's for small t. Numerical calculations show that as the squeeze parameter r is changed to 1 the super-Poissonian behaviour is persistent.

2.2 Squeezing

We study the squeezing properties of the superposition of SDFS's. It is will known the quadrature operators of the single mode field are given by

$$X_1=\frac{1}{2}(a+a^+), \qquad X_2=\frac{1}{2i}(a-a^+) \quad (2.25)$$

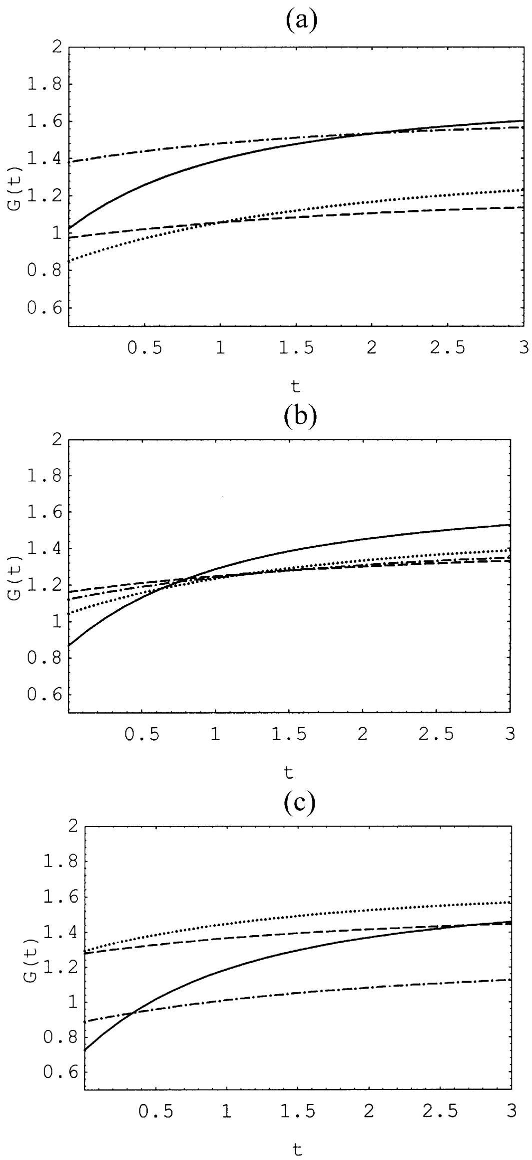

Fig. 1. Coherence function $g^{(2)}$ measured on vertical axis and horizontal axis indicates the interaction time t, with $\alpha_0 = 1$ and the squeeze parameter is assumed real and for $r = 0.5$. The amplifier parameters are $\eta = 0.2$, $\omega = 1$ and $|G| = \exp(0.2t)$. The number of photons of initial state (superposition of SDFS's) have the values; $m = 0$ (solid curve), $m = 1$ (dotted curve) $m = 2$ (chained curve) and $m = 3$ (dashed curve). The constant K is assumed as: (a) $K = 1$, (b) $K = i$ (c) $K = -1$.

such that $[X_1, X_2] = \frac{i}{2}$ which satisfies the uncertainty relation $\langle(\Delta X_1)^2\rangle\langle(\Delta X_2)^2\rangle \geq \frac{1}{16}$ with the variance $\langle(\Delta X_j)^2\rangle = \langle X_j^2\rangle - \langle X_j\rangle^2$. The field is said to be squeezed if $(\Delta X_j)^2 < \frac{1}{4}$ for ($j = 1$ or 2).

The average values of the quadrature field operators $\langle X_1\rangle$ and $\langle X_2\rangle$ are directly computed. Also variances of the quadrature field operators $\langle(\Delta X_1)^2\rangle$ and $\langle(\Delta X_2)^2\rangle$ are computed.

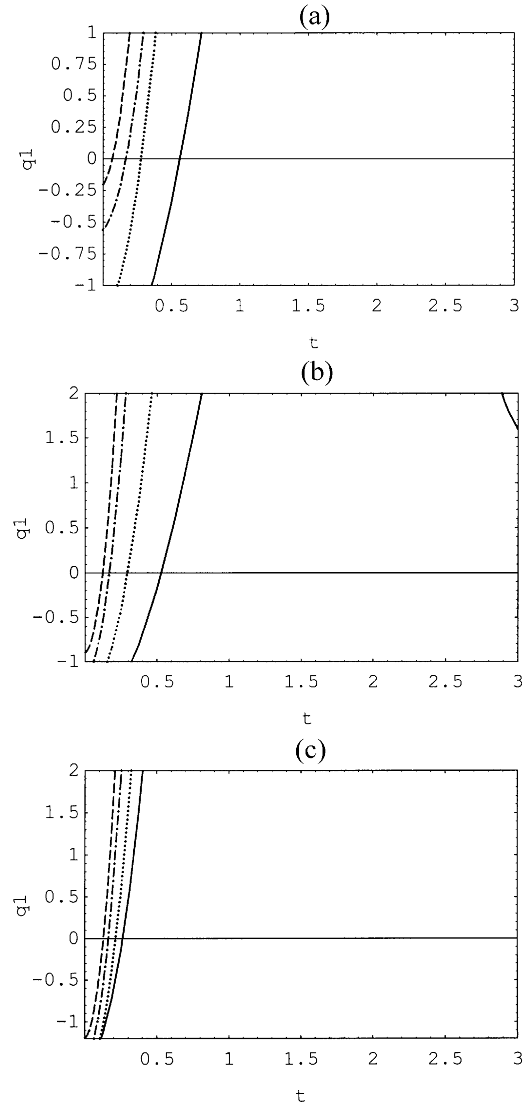

Fig. 2. Squeezing parameter q_1 against the interaction time t, with $\alpha_0 = 0.2$. The remainder parameters assume the same values in Figure 1.

The squeezing is best parameterized by

$$q_i = \frac{\langle(\Delta X_i)^2\rangle - 0.25}{0.25}, \qquad i = 1, 2 \tag{2.26}$$

such that squeezing exits for $-1 < q_i < 0$. Squeezing in one quadrature is achieved at the expense of increased noise in the conjugate quadrature; therefore, if one of q_i's is less than zero, then the other should be greater than zero.

In Figure 2 we plot q_1 against the interaction time t the squeeze with the parameter $r = 0.5$ for $\phi = 0$ and $\alpha_0 = 0.2$. The number of excitations in this superposition state are assumed $m = 0$, 1, 2, 3. The constant K has the values (a) $K = 1$, (b) $K = i$, (c) $K = -1$. It is apparent that the degree of squeezing decreases with increasing t. We find that the maximum squeezing in the case of $m = 0$ in Figure 2a,b. For $t = 0$ the increase of m implies the decrease of the squeezing degree.

$$F(\beta,s,t)=\frac{1}{\pi|G|^2A\sqrt{K_1}}\sum_{j=0}^{m}\sum_{l=0}^{j}\binom{j}{l}\binom{m}{j}\frac{(-1)^j}{(j-l)!}\left(\frac{\nu_1}{K_1}\right)^l\left(-\frac{|\nu_2|}{K_1}\right)^{j-l}$$
$$\times\left\{\exp\left[\frac{1}{K_1}(-\nu_1|\nu_3|^2+\nu_2{\nu_3^*}^2+\nu_2^*\nu_3^2)\right]H_{j-l}\left[\frac{(-\nu_1\nu_3^*+2\nu_2^*\nu_3)}{2\sqrt{(-K_1\nu_2^*)}}\right]H_{j-l}\left[\frac{(\nu_1\nu_3-2\nu_2\nu_3^*)}{2\sqrt{(-K_1\nu_2)}}\right]\right.$$
$$+|K|^2\exp\left[\frac{1}{K_1}(-\nu_1|\nu_4|^2+\nu_2{\nu_4^*}^2+\nu_2^*\nu_4^2)\right]H_{j-l}\left[\frac{(-\nu_1\nu_4^*+2\nu_2^*\nu_4)}{2\sqrt{(-K_1\nu_2^*)}}\right]H_{j-l}\left[\frac{(\nu_1\nu_4-2\nu_2\nu_4^*)}{2\sqrt{(-K_1\nu_2)}}\right]$$
$$+K\exp\left[4\Delta\{(|\mu|^2+|\nu|^2)|\bar\alpha_0|^2-\mu\nu\bar\alpha_0^{*2}-\mu\nu^*\bar\alpha_0^2\}+2\alpha_0^*\frac{\beta}{G}-2\alpha_0\frac{\beta^*}{G^*}\right]$$
$$\times\exp\left[\frac{1}{K_1}(\nu_1\nu_5\nu_6+\nu_2{\nu_6}^2+\nu_2^*\nu_5^2)\right]H_{j-l}\left[\frac{(\nu_1\nu_6+2\nu_2^*\nu_5)}{2\sqrt{(-K_1\nu_2^*)}}\right]H_{j-l}\left[\frac{(\nu_1\nu_5+2\nu_2\nu_6)}{2\sqrt{(-K_1\nu_2)}}\right]$$
$$+K^*\exp\left[4\Delta\{(|\mu|^2+|\nu|^2)|\bar\alpha_0|^2-\mu\nu\bar\alpha_0^{*2}-\mu\nu^*\bar\alpha_0^2\}-2\alpha_0^*\frac{\beta}{G}+2\alpha_0\frac{\beta^*}{G^*}\right]$$
$$\left.\times\exp\left[\frac{1}{K_1}(\nu_1\nu_7\nu_8+\nu_2{\nu_8}^2+\nu_2^*\nu_7^2)\right]H_{j-l}\left[\frac{(\nu_1\nu_8+2\nu_2^*\nu_7)}{2\sqrt{(-K_1\nu_2^*)}}\right]H_{j-l}\left[\frac{(\nu_1\nu_7+2\nu_2\nu_8)}{2\sqrt{(-K_1\nu_2)}}\right]\right\}\quad(3.1a)$$

Numerical calculations show that as the α_0 decreases the squeezing degree of q_i increases. Also the maximum degree of the second-order squeezing of q_2 exist in the interval $1.2 < t < 2$. We can conclude that the output of linear amplifier with superposition of pair of SDFS's as initial state exhibits different nonclassical effects which depend on the particular choice of the phases θ_0 and ϕ.

3 s-parameterized quasi-probability function

Quasi-probability distribution functions, [3,4] such as Glauber's P function, the Wigner W function and the Q function proved to be very useful theoretical tools in performing quantum optical calculations. These functions provide a way to characterize the non-classical nature of a quantum field. They have now actually become accessible to measurements [5].

The s-ordered distribution functions are defined as a Fourier transformation of the s-ordered CF, and can be obtained by using (2.15) in (2.2). By performing the integration [1b], then the s-ordered distribution function for the output linear amplifier field may be written in the form:

See equation (3.1a) above

where

$$\nu_1=\frac{\{1-s+2M(t)\}(|\mu|^2+|\nu|^2)}{2|G|^2}-|\nu|^2,\quad(3.1b)$$

$$\nu_2=\{1-s-|G|^2+2M(t)\}\frac{\mu\nu^*}{2|G|^2},\quad(3.1c)$$

$$\nu_3=\mu\left(\frac{\beta^*}{G^*}-{\alpha_0}^*\right)+\nu^*\left(\frac{\beta}{G}-\alpha_0\right),\quad(3.1d)$$

$$\nu_4=\mu\left(\frac{\beta^*}{G^*}+{\alpha_0}^*\right)+\nu^*\left(\frac{\beta}{G}+\alpha_0\right),\quad(3.1e)$$

$$\nu_5=\left[2\Delta(|\mu|^2+|\nu|^2)\bar\alpha_0^*-4\Delta\mu\nu^*\bar\alpha_0-\left(\mu\frac{\beta^*}{G^*}+\nu^*\frac{\beta}{G}\right)\right],\quad(3.1f)$$

$$\nu_6=\left[2\Delta(|\mu|^2+|\nu|^2)\bar\alpha_0-4\Delta\mu\nu\bar\alpha_0^*+\left(\mu\frac{\beta}{G}+\nu\frac{\beta^*}{G^*}\right)\right],\quad(3.1g)$$

$$\nu_7=\left[-2\Delta(|\mu|^2+|\nu|^2)\bar\alpha_0^*+4\Delta\mu\nu^*\bar\alpha_0-\left(\mu\frac{\beta^*}{G^*}+\nu^*\frac{\beta}{G}\right)\right],\quad(3.1h)$$

$$\nu_8=\left[-2\Delta(|\mu|^2+|\nu|^2)\bar\alpha_0+4\Delta\mu\nu\bar\alpha_0^*+\left(\mu\frac{\beta}{G}+\nu\frac{\beta^*}{G^*}\right)\right],\quad(3.1i)$$

$$\Delta=\frac{s-1}{2}+\frac{1}{2}|G|^2-M(t)\quad(3.1j)$$

$$\text{and}\qquad K_1=\nu_1^2-4|\nu_2|^2\quad(3.1k)$$

From this formula the exact analytical expressions for the s-parameterized quasiprobability distribution function for the output linear amplifier with the superposition of the coherent states [3], squeezed states [7] and displaced Fock

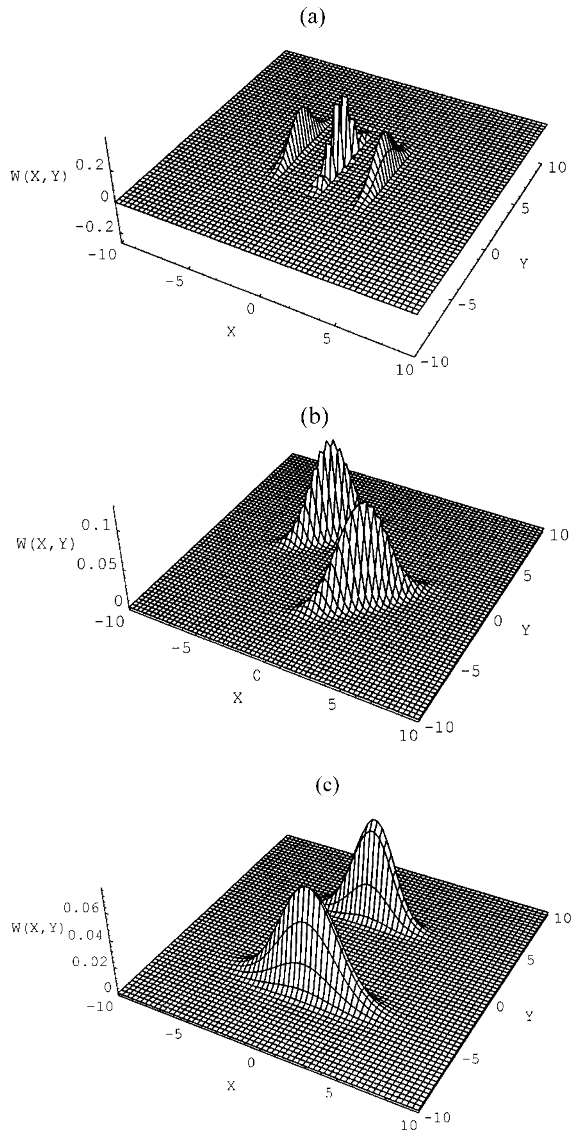

Fig. 3. Three-dimensional time dependence of a Wigner distribution function for the output of the linear amplifier driven by the SDFS's superposition with $m = 0$, $\alpha_0 = 2$, and squeeze parameter $r = 1$, and its direction $\phi = 0$. The amplifier parameters assume the same values in Figure 1. Here $X = \mathrm{Re}(\beta)$ and $Y = \mathrm{Im}(\beta)$. The interaction time $t = \frac{n\pi}{4}$, $n = 0$, 1, 2, is chosen for illustration.

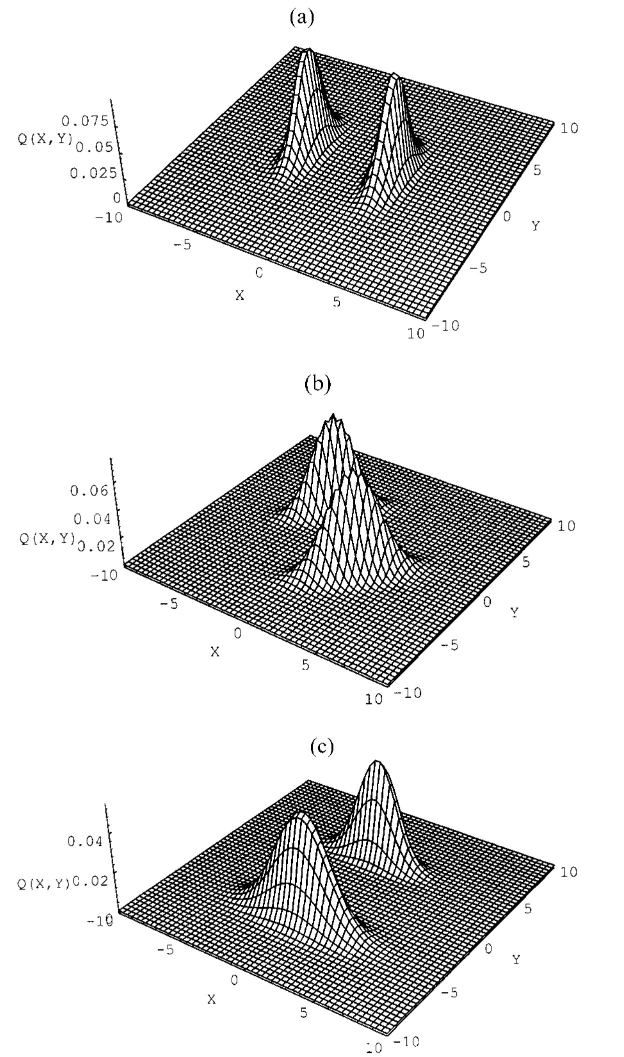

Fig. 4. The Q-function (*i.e.*, $s = -1$) for the output of the linear amplifier driven by the SDFS's superposition state with the same parameters in Figure 3.

states [8] can be found as special cases. We note that The P function, *i.e.*, $s = 1$ exists for the output linear amplifier with the SDFS's superposition states, when $t > \frac{\pi}{2}$.

In Figure 3 we plot the Wigner function, *i.e.*, $s = 0$ with the parameters having the values: $m = 0$, *i.e.*, for the squeezed state superposition, $\alpha_0 = 2$, $r = 1$, $\phi = 0$ and $K = 1$. The amplifier parameter is $|G| = \exp(0.2t)$. The interaction time $t = \frac{n\pi}{2}$, $n = 0$, 1, 2, is chosen for illustration. It is clear that (at $t = 0$) from the form of the Wigner function the three peaks observed may be easily found at $x = \pm\alpha_0$, 0 and $y = 0$. The nonclassical nature of the superposition of two SDFS's is indicated by the negative values of the Wigner distribution function. As time increases we observe that the function rotates in the phase space and it spread out with flattening of middle peak.

In Figure 4 we have the Q function for the output linear amplifier with the squeezed state superposition (*i.e.*, $m = 0$) as an input state with the same parameters as in Figure 3. We note that for $\alpha_0 < 1$ the Q function for $t = 0$ is a Gaussian centred at the origin. As α_0 increases we see that the Gaussian splits into two peaks moving along the positive and negative x-axis, with greater separation for increasing of α_0. It is easily found that the two peaks at $x = \pm\alpha_0$ and $y = 0$. Similar behaviour is infered from Figures 3, 4 as t develops.

The Wigner functions of the output linear amplifier with the pair of SDFS's superposition state as input for $m = 1$, $\alpha_0 = 2$ and $z = 1$ are shown in Figure 5. The amplifier parameters have the same values as in Figure 3. From the plots, two separated negative peaks and an oscillatory regime between them can be seen at $t = 0$.

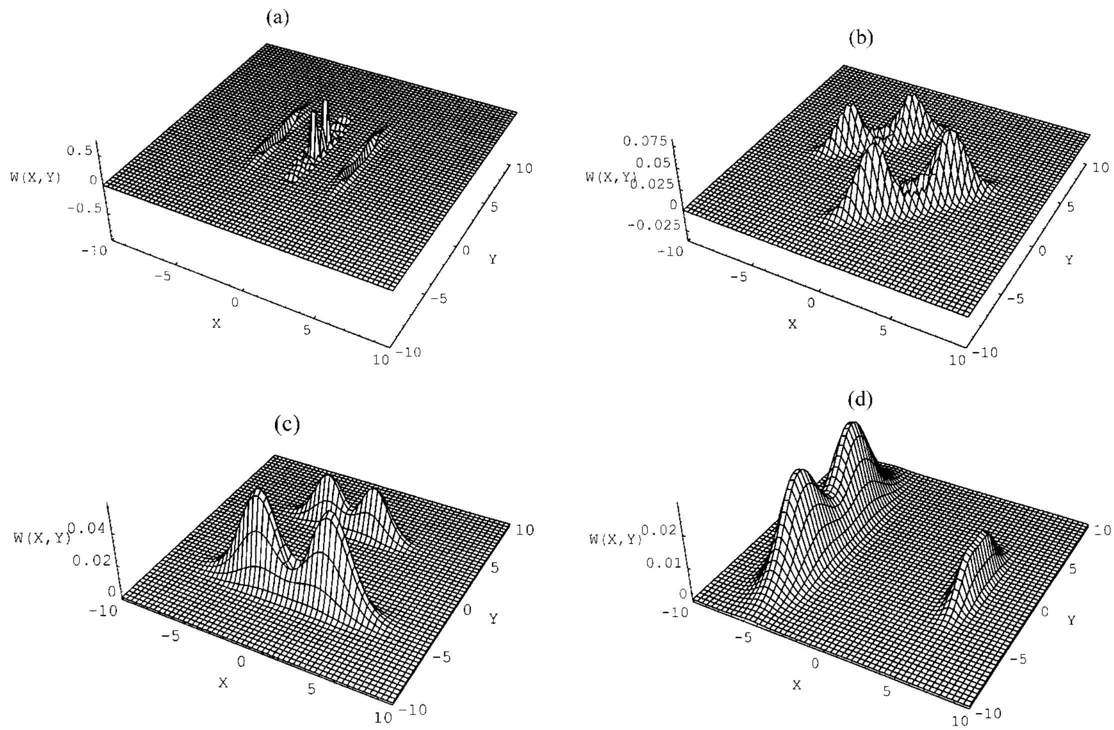

Fig. 5. Temporal behaviour of a Wigner distribution function for the output of the linear amplifier driven by the SDFS's superposition with $m = 1$, $\alpha_0 = 2$, $r = 1$, and $\phi = 0$. The amplifier parameters assume the same values in Figure 1. Here $X = \mathrm{Re}(\beta)$ and $Y = \mathrm{Im}(\beta)$. The interaction time $t = \frac{n\pi}{4}$, $n = 0, 1, 2, 4$, is chosen for illustration.

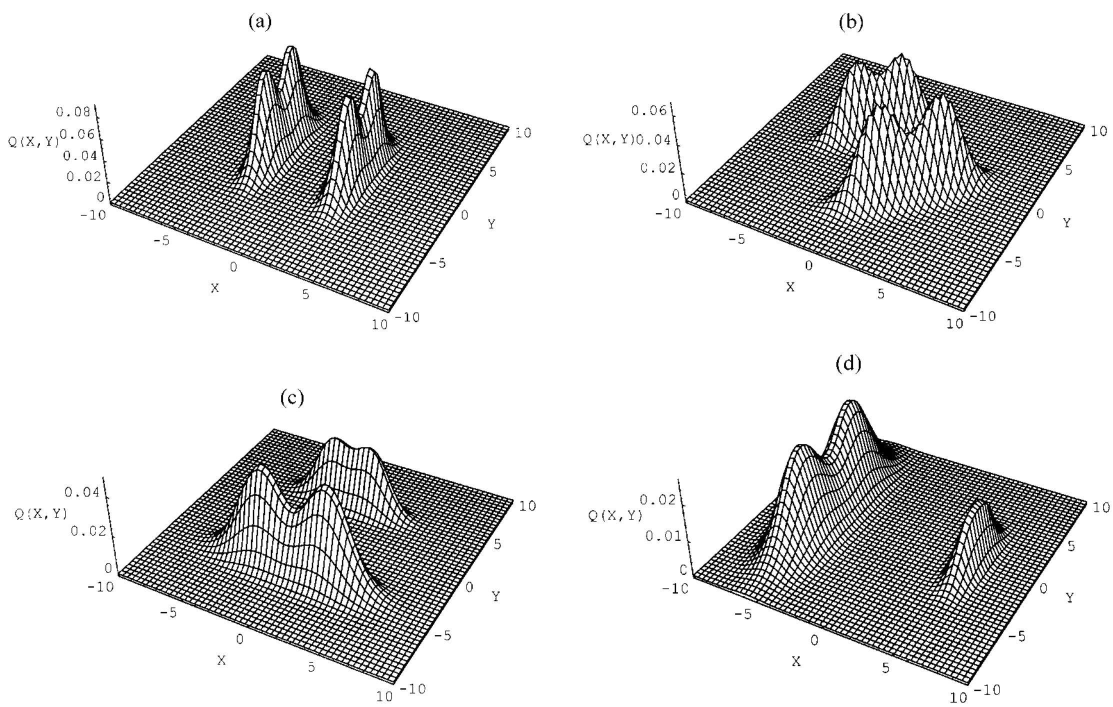

Fig. 6. Temporal behaviour of the Q-function for the output of the linear amplifier driven by the SDFS's superposition state with the same parameters in Figure 5.

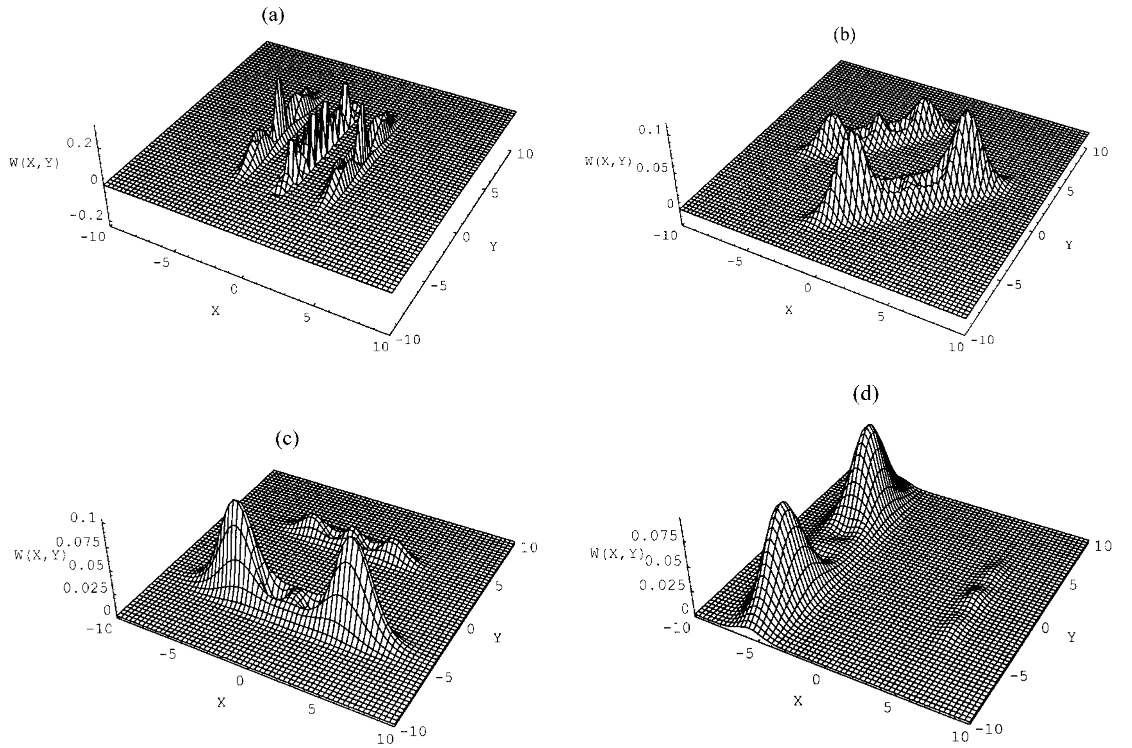

Fig. 7. The W (Wigner) function (*i.e.*, $s = 0$) for the output of the linear amplifier driven by a SDFS's superposition state with $m = 2$, $\alpha_0 = 3$, $r = 1$, and $\phi = 0$. The amplifier parameters are the same as in Figure 1. The interaction times $t = \frac{n\pi}{4}$, $n = 0$, 1, 2, 4 are chosen to illustrate the dependence on time and the rotation in the phase space. Here $X = \mathrm{Re}(\beta)$ and $Y = \mathrm{Im}(\beta)$.

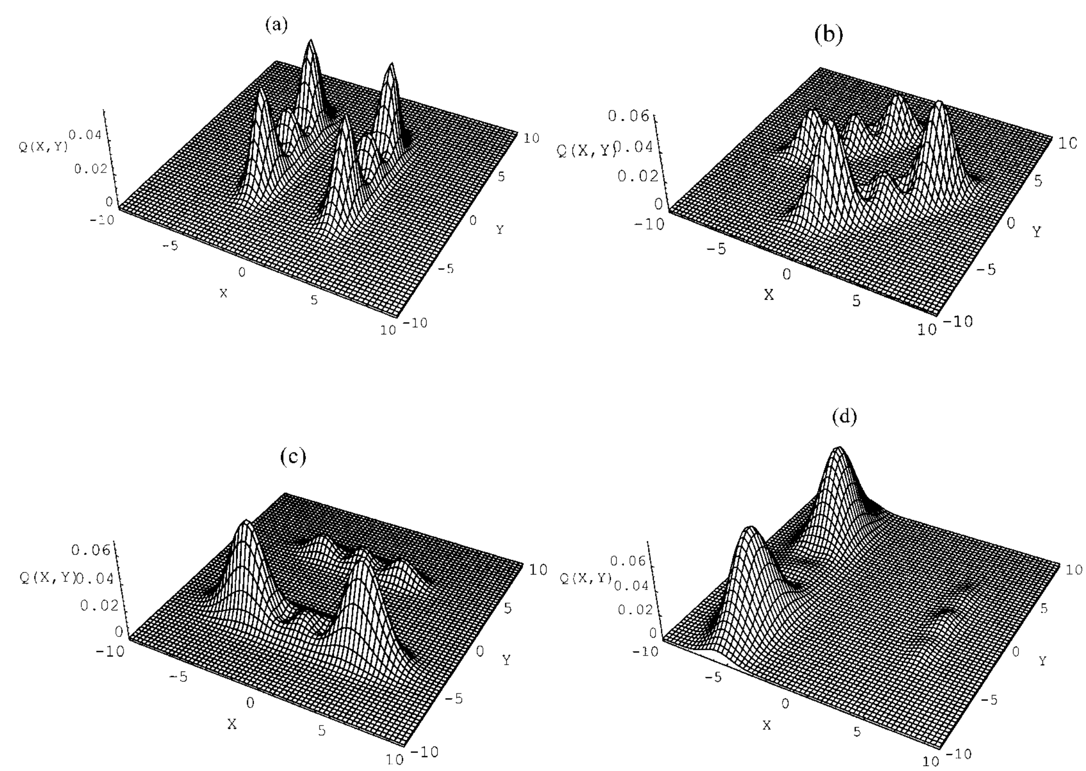

Fig. 8. Temporal behaviour of the Q-function for the output of the linear amplifier driven by the SDFS's superposition state with the same parameters in Figure 7.

The separation of the two peaks is seen to increase with α_0, but the oscillatory regime increase the with increase of r. The constant $K = 1$, and the interaction time $t = \frac{n\pi}{2}$, $n = 0, 1, 2, 4$, are chosen for illustration.

In Figure 6 we show the plots of the Q function with the same parameters as in Figure 5. Note that as time increases one f side of the figure diminishes in a faster way than the other giving an asymmetry and the effect of superposition s almost lost.

In Figure 7 we plot the Wigner function of the output linear amplifier with the pair of SDFS's superposition state as input with $m = 2$, $\alpha_0 = 3$, $z = 1$, $K = 1$ and t as in Figure 5. The amplifier parameters have the same values as in Figure 3. We note that the oscillatory regime between the two peaks increased with the increase of m at $t = 0$. The Q function is shown in Figure 8 with the same parameters as those of Figure 7. We note that for chosen α_0 the squeezing effect decreases with increasing m. From these figures it is seen that asymmetry starts to develops as t increases.

Generally it is seen that, for $t = 0$ behaviour of Wigner and Q functions exhibits the standard distributions of the pair of SDFS's superposition state as shown in reference [15]. With increasing of time the maximum values of Wigner and Q functions decrease and rotate in a clockwise direction. This means a loss of squeezing as time develops. The rotation in the phase space is due to the appearance of the frequency in the factor G. The spreading and shrinking of Wigner and Q functions over the β-plane is shown as time advances. The flattening of the two peaks are shown as time advances, which means an increase of diffusion as interaction time t progresses. As t becomes greater than 2π the the various quasiprobability functions (*i.e.*, P, Wigner and Q functions) behave in nearly the same way, and they almost have the same shape.

4 Conclusion

We have discussed the s-ordered characteristic function and quasiprobability distribution function for the output linear amplifier with the pair of SDFS's superposition state as input. We have obtained the formulae for the s-ordered characteristic function for the output linear amplifier with the pair of SDFS's superposition as an initial field. Several moments have been calculated by using the characteristic function as a function of the interaction time. The second-order correlation function $g^{(2)}$ has been investigated numerically. The squeezing properties for these states have been discussed. The Wigner and Q functions for some parameters have presented analytically and numerically. We have demonstrated the rotation of the quasiprobability distribution function in the phase space as a function of interaction time. It has also been exhibited the asymmetrical diffusion for superposition states. Our results generalize these in [1,2] for the linear amplifier. The physical interpretation of the output of linear amplifier with superposition of SDFS's as input tends to that for SDFS's input as we have shown [1b] while time greater than π.

Our present work was motivated by the desire to realize physically certain specific quantum states (superposition of SDFS's [15]) and use it as input for the linear-insensitive amplifier as one of its applications. It is hoped that the superposition of SDFS's will find application in the quantum nondemolition measurements and quantum optics. They may also find applications in experimental situations that require low noise sensitivity.

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